Dark Matter Theory, Simulation, and Analysis in the Era of Large Surveys, KITP UCSB

How well do we have gravothermal phases in SIDM halos?

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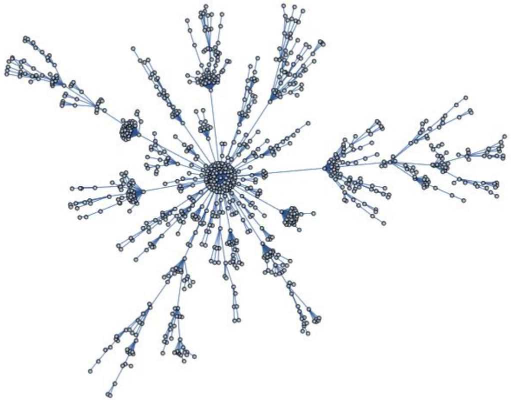
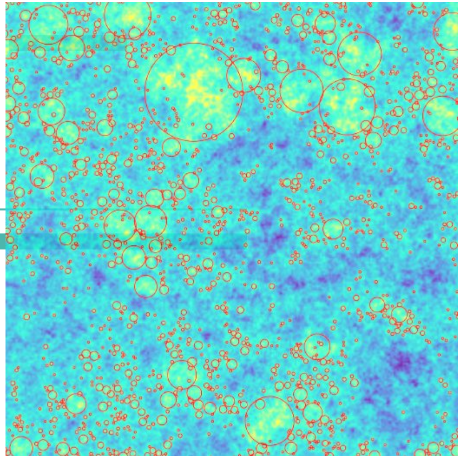


<<<- Graphs made during the workshop ->>>



A break down of the question

$$\tau \equiv t/t_c$$

1. How well is tc being proportional to sigma/m?
2. How does accretion history modify the gravothermal phase?
3. Does gravothermal phase capture all the SIDM information? (DM-only)
4. How do baryons boost the gravothermal evolution?
5. How does baryon growth change the gravothermal phase?
6. Does gravothermal phase capture all the SIDM information? (add baryons)

Answering these questions lead us to a parametric model
Detailed in papers with Ethan O. Nadler, Hai-Bo Yu,
Yi-Ming Zhong, S. Ando, S. Horigome

$$\begin{aligned}\rho_s(\tau) &= \rho_{s,0}\, g_\rho(\tau)\\ r_s(\tau) &= r_{s,0}\, g_r(\tau)\\ r_c(\tau) &= r_{s,0}\, g_c(\tau)\end{aligned}$$

SIDM leads to a **self-gravitating** & **thermalizing** system

Core formation

- energy flux + capacity => core formation

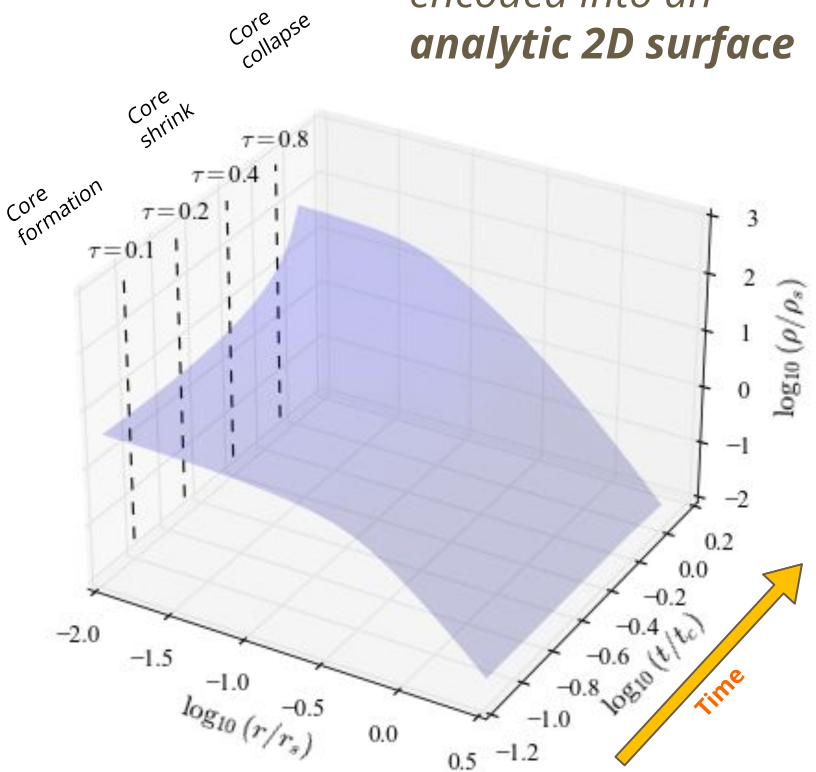
Core shrink

+ energy flux + capacity => quasi-stable core

Second stage

+ energy flux **- capacity** => **core collapse**

Major features of this evolution can be encoded into an ***analytic 2D surface***



Kaplinghat, Tulin, and Yu 1508.03339
Yang & Yu 2205.03392
and many more...

A “clock” in the gravothermal evolution: Gravothermal phase

SIDM generates an arrow of time

Normalized to give a “clock” / “phase”

$$\frac{\partial}{\partial r}\left(r^2 \kappa m \frac{\partial \nu^2}{\partial r}\right) = r^2 \rho \nu^2 \frac{D}{\boxed{Dt}} \ln \frac{\nu^3}{\rho}$$

When **κ** ∝ # of scatterings ∝ **σ** (long-mean-free-path regime)

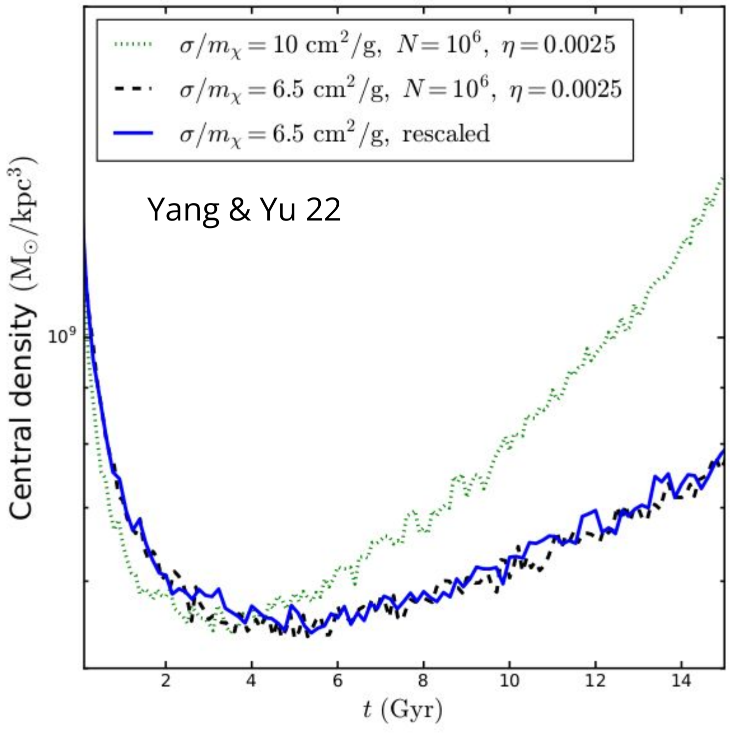
The **cross section (σ)** dependence can be absorbed into the **arrow of time as: t -> t σ**

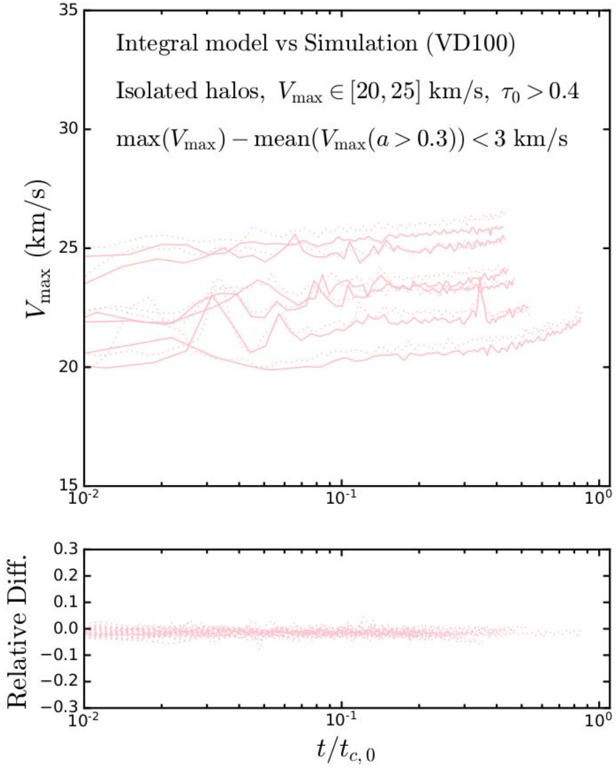
$$\tau \equiv t/t_c$$

SIDM independence キ unitless

Related discussion in the context of “universality”: Outmezguine+ 2204.06568; Yang+ 2305.16176; Zhong+2306.08028; Yang2405.03787

1. How well is tc being proportional to sigma/m?





"Isolated" halos in cosmological simulation (Yang, Nadler, Yu 23) with

- t/tc>0.4
- Exclude cases with major mergers

Model vs simulation

More in YNY24 to appear

2. How does accretion history modify the gravothermal phase?

Step 1: a fictitious CDM halo

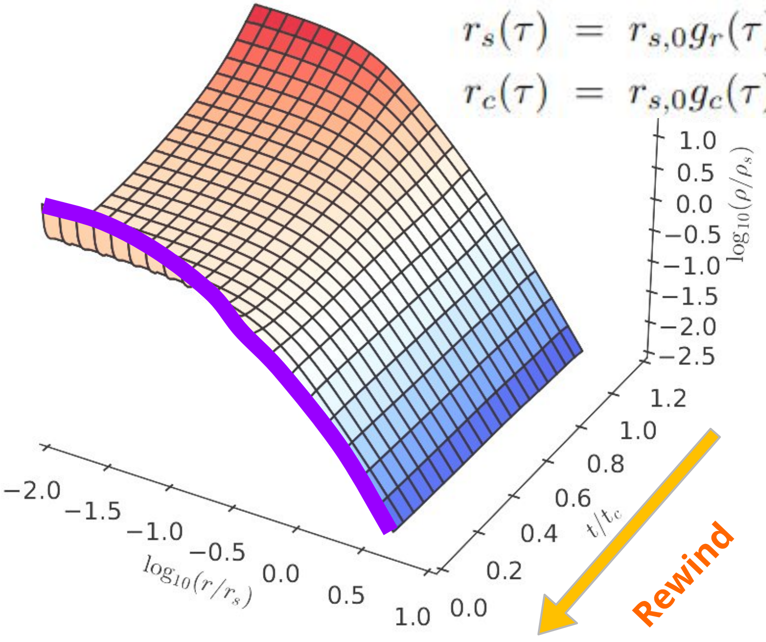
Rewinding the gravothermal phase would result in a **fictitious** CDM halo

Assuming that all the SIDM effect in an *isolated* halo is captured by the phase:

Fictitious CDM halo == Simulated CDM halo

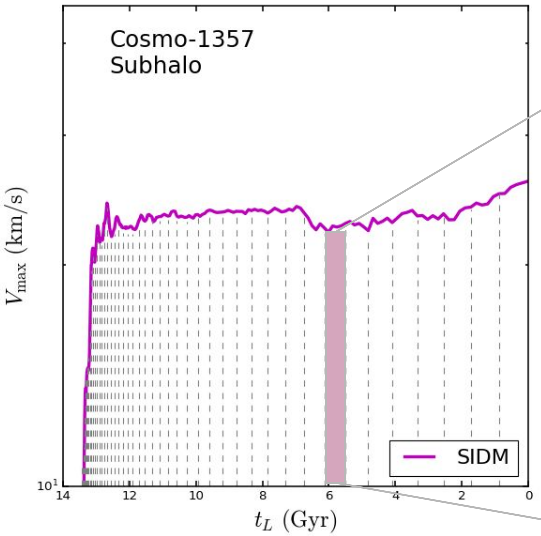
In reality, a halo has an **accretion history**, which may change both the phase and the fictitious CDM halo

$$\rho_s(\tau) = \rho_{s,0}\, g_\rho(\tau)$$
$$r_s(\tau) = r_{s,0}\, g_r(\tau)$$
$$r_c(\tau) = r_{s,0}\, g_c(\tau)$$



t/tc=0 corresponds to an **NFW profile**

Step 2: gravothermal phase from a population of fictitious CDM halos



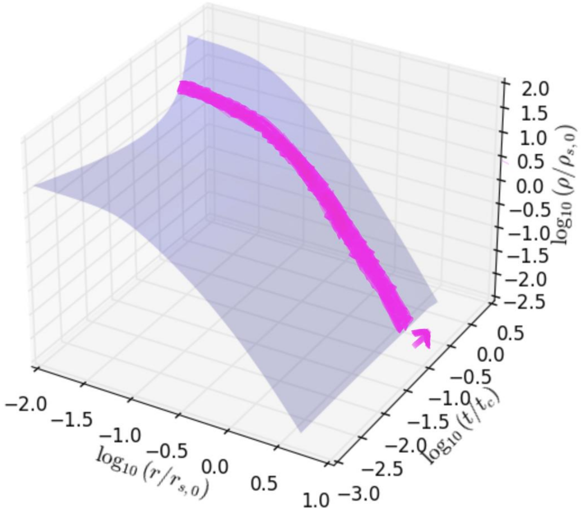
For **every** small time increment, the **Δτ** can be computed using the **fictitious CDM halo**

Example: τ=t/tc≈0.6

In Δt=0.5 Gyr, almost no mass change. Then the increment in gravothermal phase is:

Δτ=(Δt)/tc

where tc is computed using the fictitious CDM halo params



arXiv:2305.16176

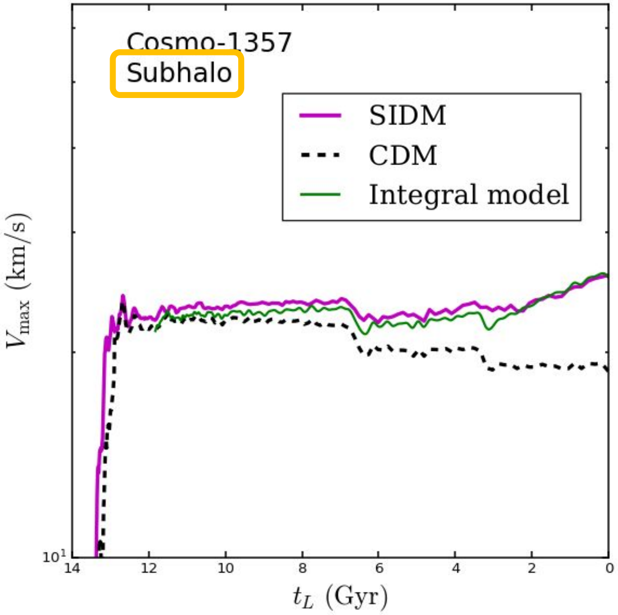
The integral approach

Model (Integral) prediction agree with the SIDM simulation well:

Gravothermal phase and its increments successfully capture the leading effects of SIDM halo evolution

Applicability

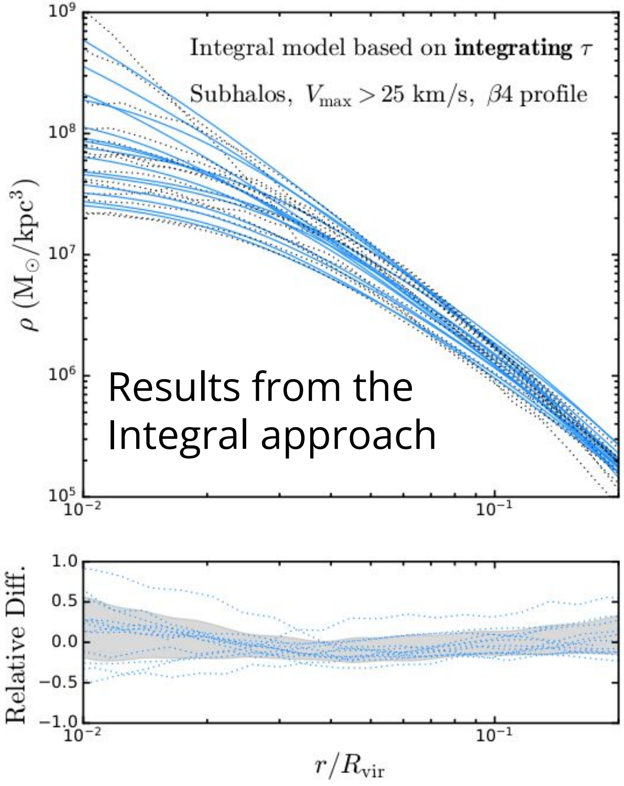
- Small Δτ during the Δt of a merger
- CDM halo mass close to SIDM halo mass at all times



$$V_{\max}(t) = V_{\max,\mathrm{CDM}}(t) + \int_0^{\tau(t)} d\tau' \frac{dV_{\max,\mathrm{Model}}(\tau')}{d\tau'}$$

$$R_{\max}(t) = R_{\max,\mathrm{CDM}}(t) + \int_0^{\tau(t)} d\tau' \frac{dR_{\max,\mathrm{Model}}(\tau')}{d\tau'}$$

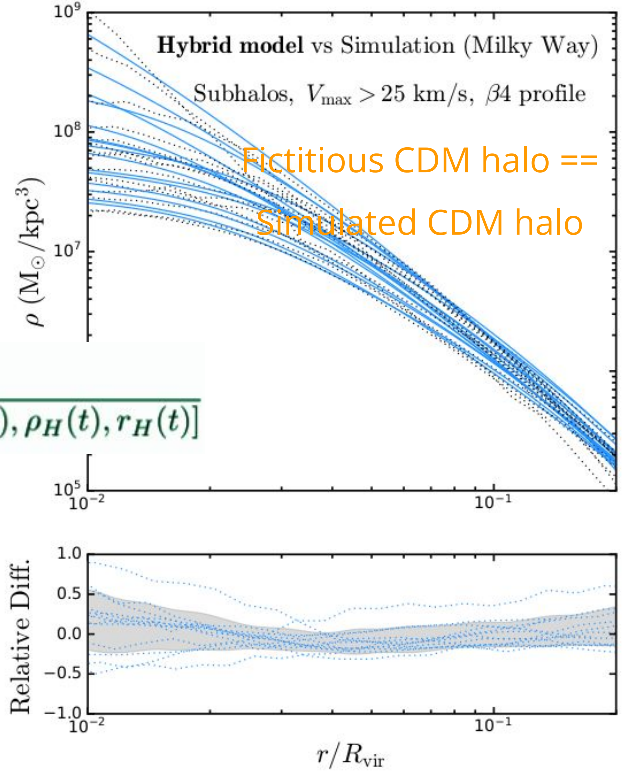
3. Does gravothermal phase capture all the SIDM information?



Results from the Integral approach

What if we just use the CDM halo from simulation?

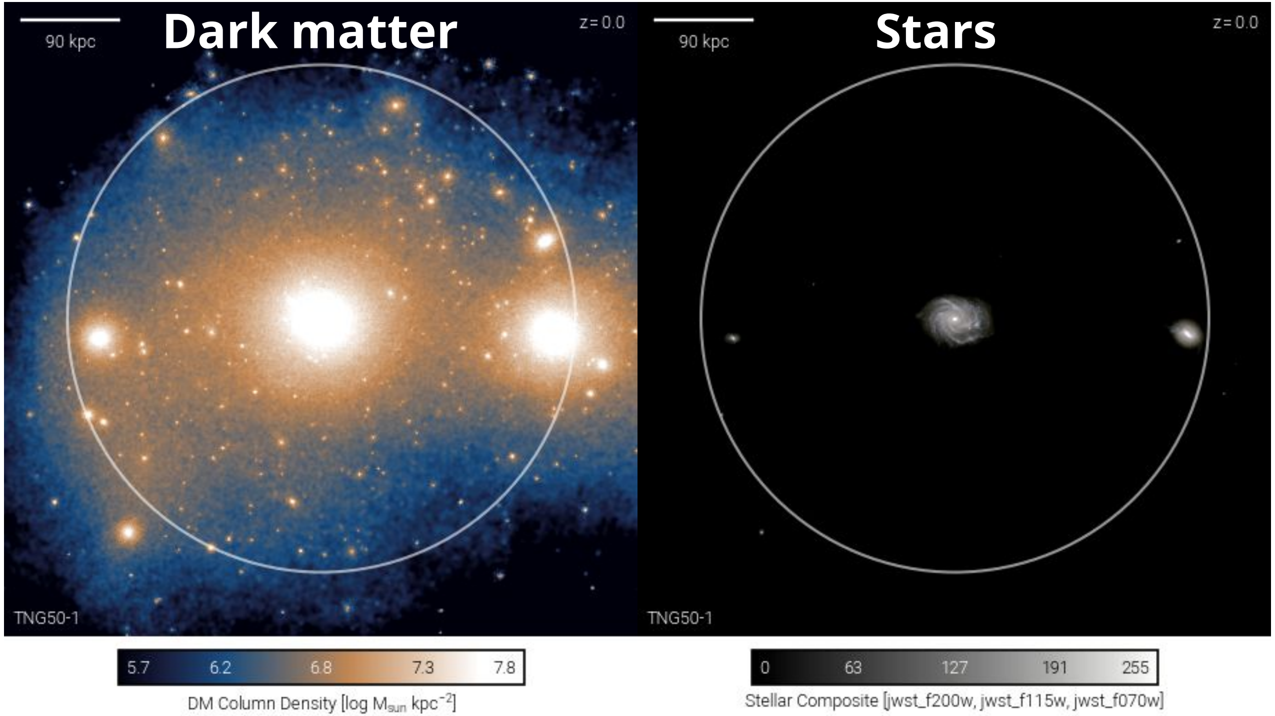
$$\tau(t) = \int_0^t \frac{dt}{t_{c,b}[\sigma_{\rm eff}(t)/m, \rho_s(t), r_s(t), \rho_H(t), r_H(t)]}$$



Fictitious CDM halo == Simulated CDM halo

Similar performance
In the DM-only case

SIDM effect is most prominent in the central region, where baryons populate



Figs. credit: TNG collaboration

4. How do baryons boost the gravothermal evolution?

A new equation for the core collapse time

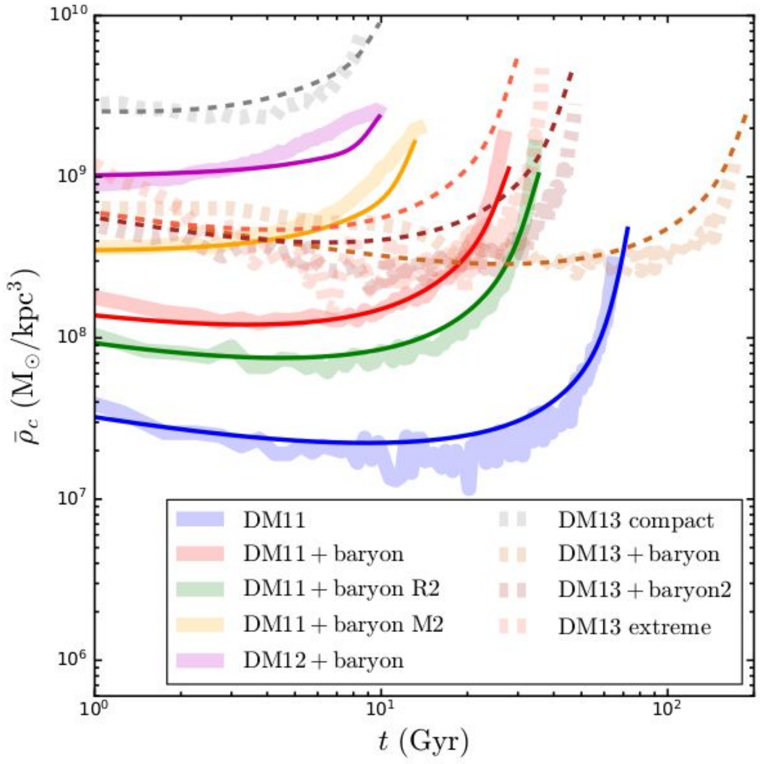
- Based on energy transfer
- Incorporate baryons
- *Collision rate => Energy transport*

$$t_R(r) \propto \frac{M(r)|\Phi(r)|/2}{4\pi r^2 \kappa |\nabla T|}$$

A density profile that allows incorporating adiabatic contraction effect

$$\rho_{\rm CoredDZ}(r) = \frac{f_{\rm in}(r)\rho_x f_{\rm out}(r)}{\frac{(r^k+r_c^k)^{1/k}}{r_x}\left(1+\left(\frac{r}{r_x}\right)^{1/2}\right)^{2(3.5-a)}}$$

N-body simulation: bands
Parametric model: lines



arXiv:2405.03787

4. How do baryons boost the gravothermal evolution?

$$t_{c,b} = t_{c,0}\mathcal{F}_t(\hat{\rho}_H, \hat{r}_H)$$

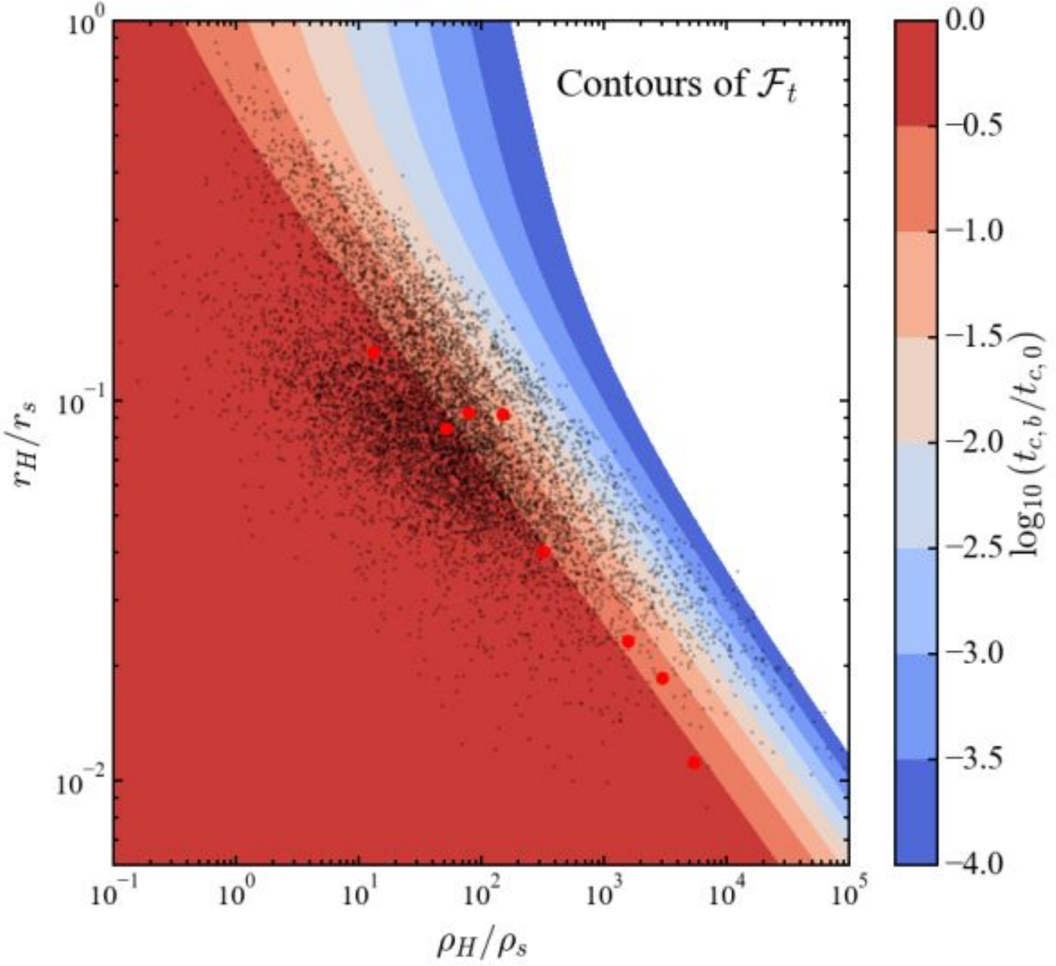
The ratio of core collapse times w&o baryons

$$\mathcal{F}_t = \left(\frac{1}{\hat{r}_{\rm eff}} + \frac{\gamma\hat{\rho}_H\hat{r}_H^3}{\hat{r}_{\rm eff}(\hat{r}_{\rm eff}+\hat{r}_H)^2}\right)^{-1}\left(1+\alpha\frac{\hat{\rho}_H\hat{r}_H^2}{2}\right)^{-\frac{1}{2}}$$

Lower-left: marginal effect

Upper-right: can be orders of magnitude large

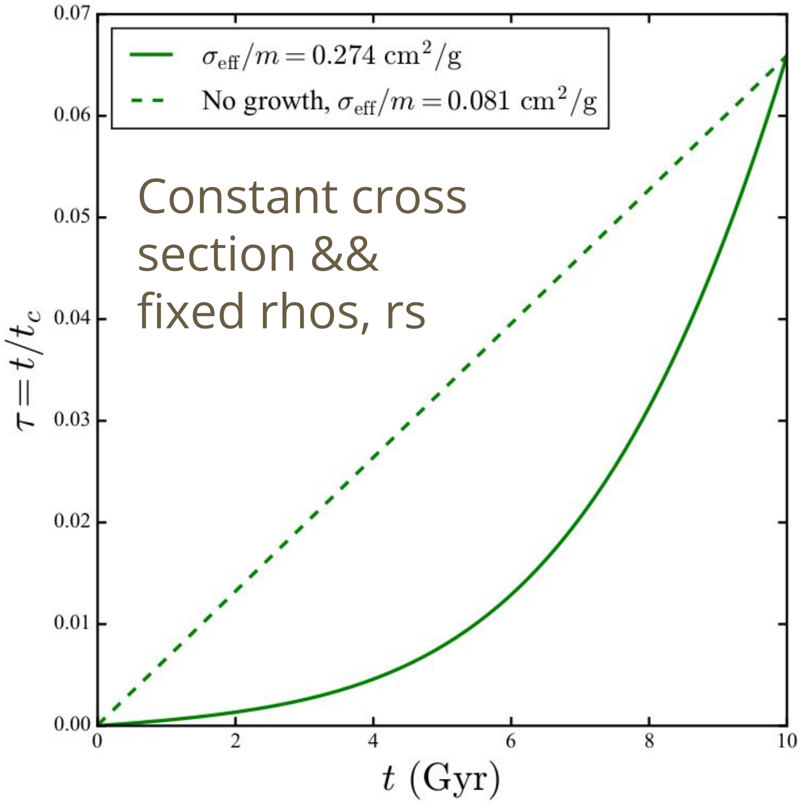
ρH, rH: Hernquist scale radius and density of the stellar component



5. How does baryon growth change the gravothermal phase?

$$\tau(t) = \int_0^t \frac{dt}{t_{c,b}[\sigma_{\rm eff}(t)/m, \rho_s(t), r_s(t), \rho_H(t), r_H(t)]}$$

One example: Incorporating baryon growth, the resulting SIDM model given the same profile is modified by (0.274-0.081)/0.081=238%



6. Does gravothermal phase captures all the SIDM information?

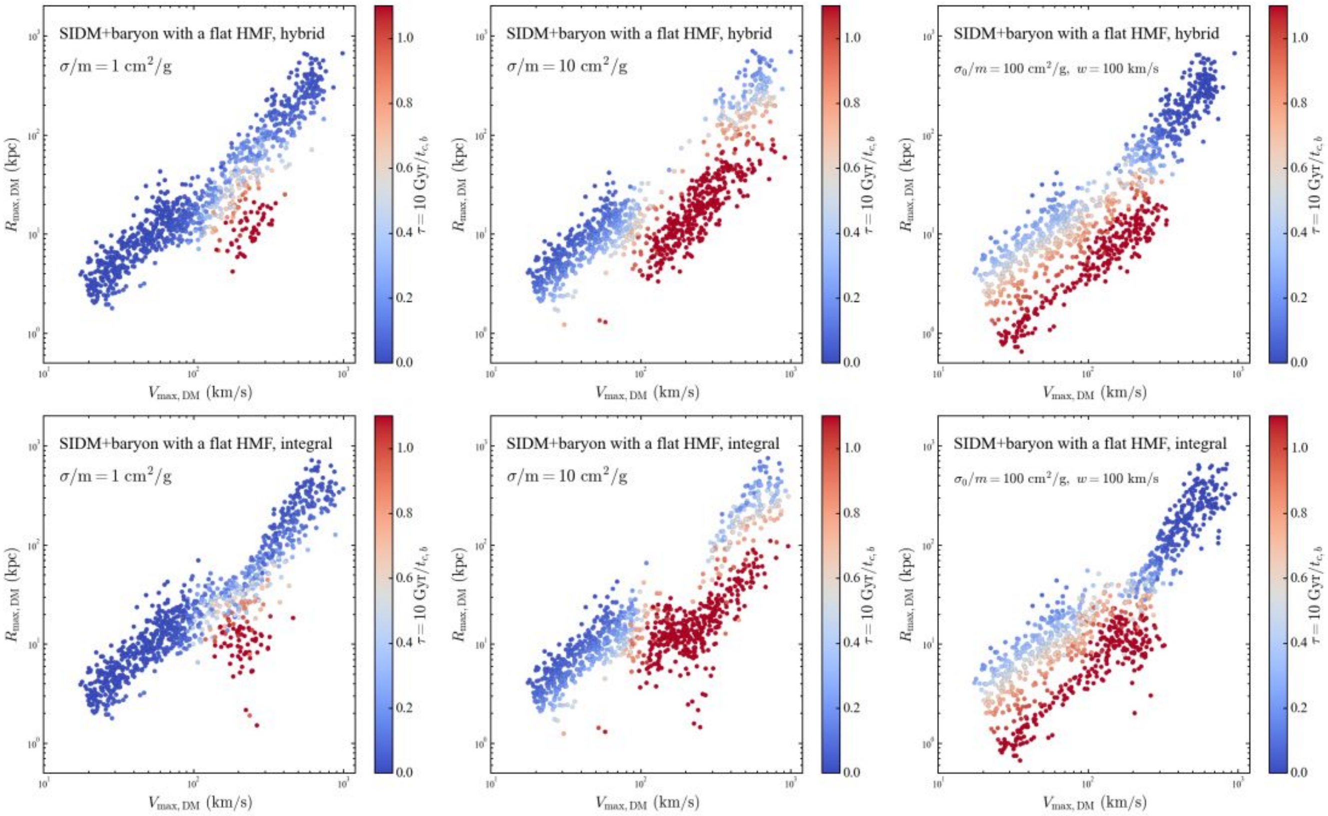
- *For DM-only simulations, the fictitious CDM halo is very close to the halo in the CDM simulation*
- *In the presence of baryons, the difference will increase*

6. Does gravothermal phase captures all the SIDM information? (add baryons)

Not really, but the difference is largely buried after including the baryons

Vmax & Rmax from **DM only**

Upper panels: CDM params @z=0 + gravothermal phase



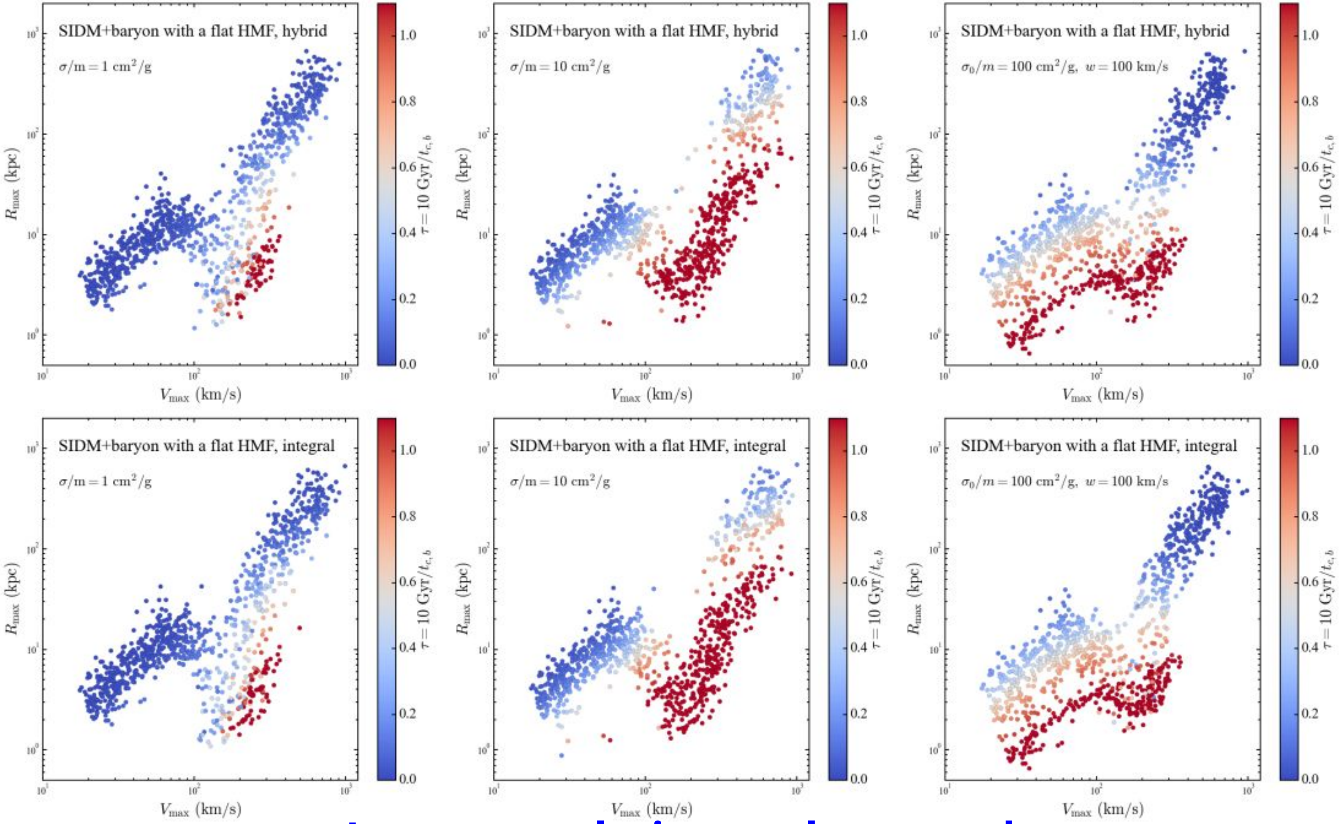
Lower panels: integral approach

6. Does gravothermal phase captures all the SIDM information? (add baryons)

Not really, but the difference is largely buried after including the baryons

Vmax & Rmax from **DM+baryons**

Upper panels: CDM params @z=0 + gravothermal phase



Lower panels: integral approach

[There are more questions…](#)

Summary

1. How well is tc being proportional to sigma/m?

- **Good enough for enable a parametric model for SIDM halos**

2. How does accretion history modifies the gravothermal phase?

- **Largely through the integral approach**

3. Does gravothermal phase captures all the SIDM information? (DM-only)

- **Almost yes, in the DM-only case**

4. How does baryons boost the gravothermal evolution?

- **A broad spectrum which can be quantified**

5. How does baryons growth change the gravothermal phase?

- ***Can use the integral approach***

6. Does gravothermal phase captures all the SIDM information? (add baryons)

- **Not really, but it still captures the majority of the information**

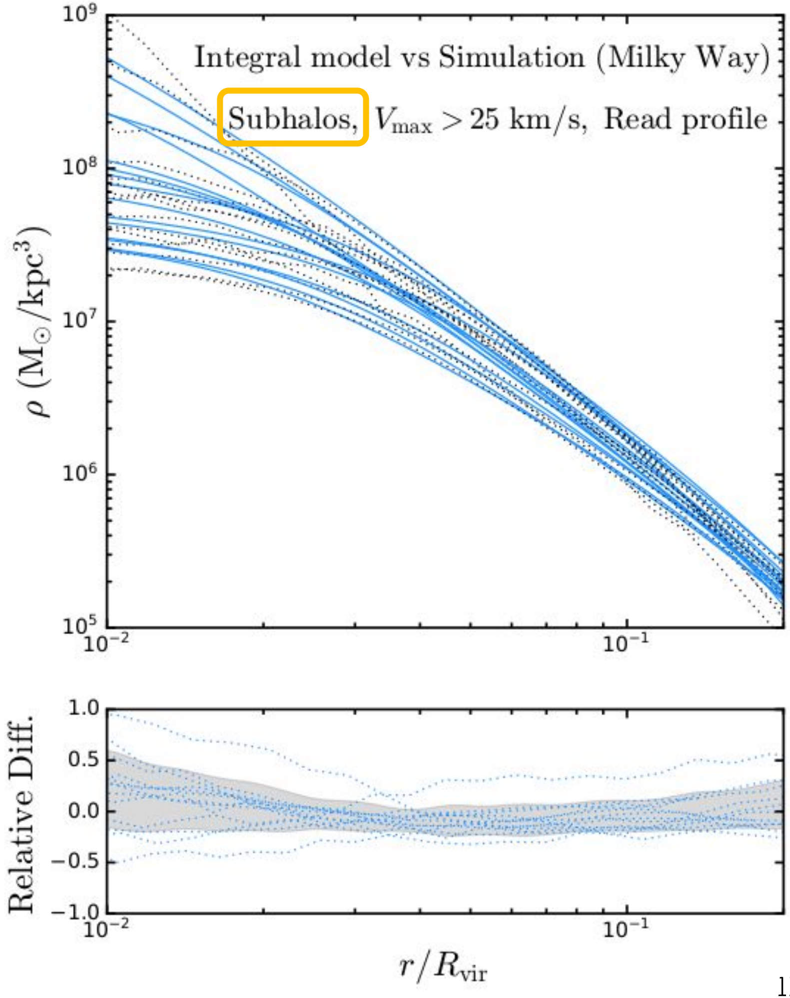
Parametric analysis tools for SIDM halos

An efficient tool for obtaining SIDM predictions

- Based on a few analytic functions/trajectories of the *gravothermal phase*
- Grounded in theory principles: not just an empirical model
- Tested against a large number of halos in cosmological simulations
- Has been extended to incorporate *mass changes* and *baryon potentials* arXiv:2405.03787

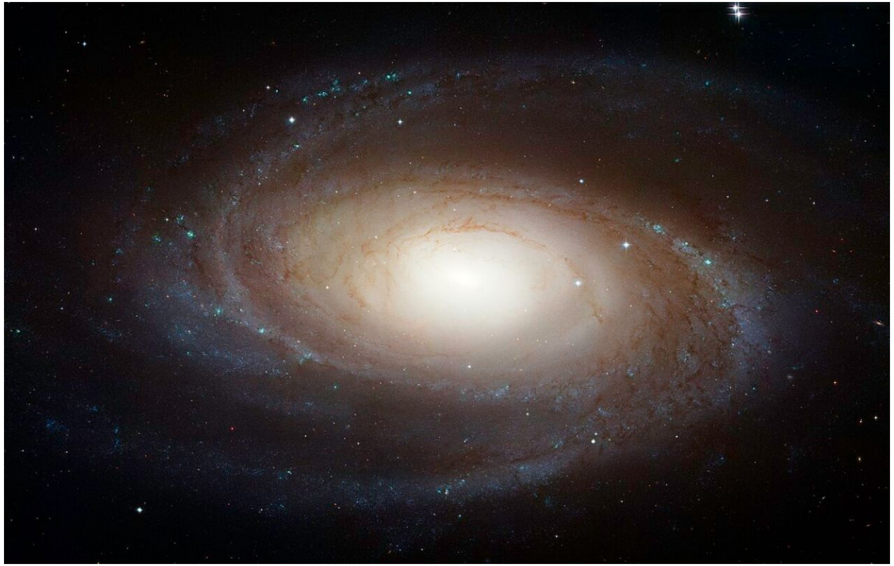
https://github.com/DanengYang/parametricSIDM

With Ethan O. Nadler, Hai-Bo Yu, Yi-Ming Zhong, S. Ando, S. Horigome



Backup

Examples applications



M81 (Milky Way like)

Ms=6.38e10 Msun;

Diameter=28.4 kpc

Re~7 kpc

Hernquist params rH=3.92 kpc

rhoH=Ms/(2*pi*rH^3)=1.686e8

*MBH=7e7 Msun

IC2574 DM dominated

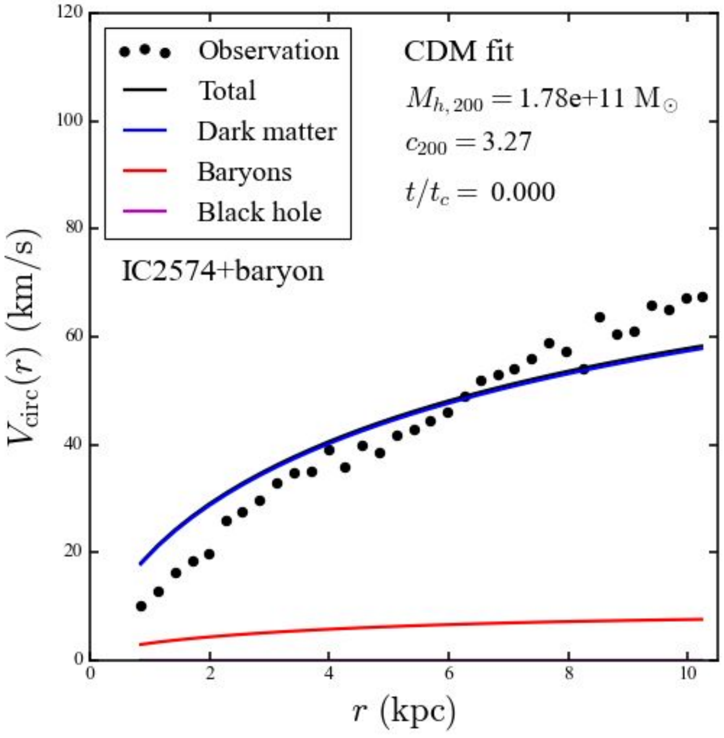
Re=3.18 kpc

Mb=5.08e8 Msun

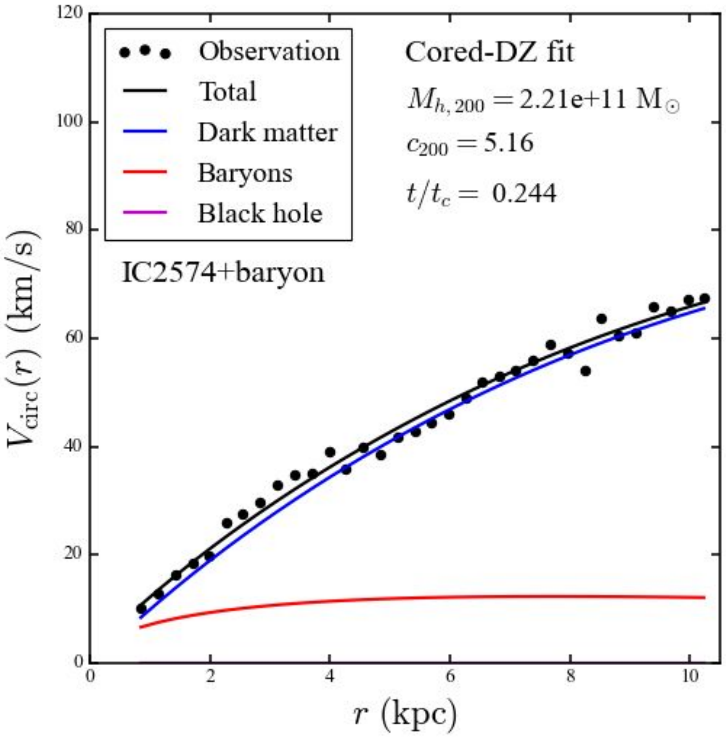
rH=1.317 kpc

rhoH= 3.5378e7 Msun/kpc^3

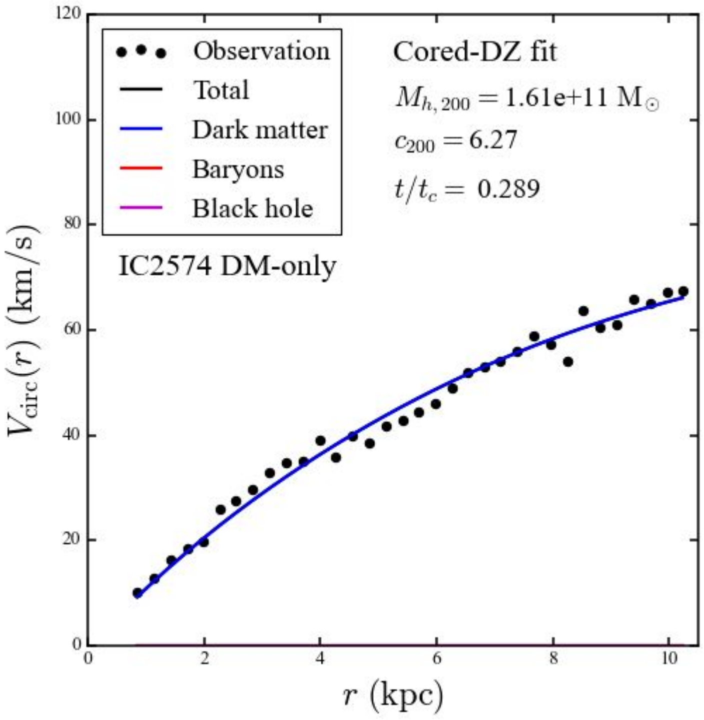
CDM


Observation
Total
Dark matter
Baryons
Black hole
CDM fit
$M_{h,200} = 1.78e{+}11\ M_\odot$
$c_{200} = 3.27$
$t/t_c = 0.000$
IC2574+baryon
$V_{circ}(r)$ (km/s)
r (kpc)

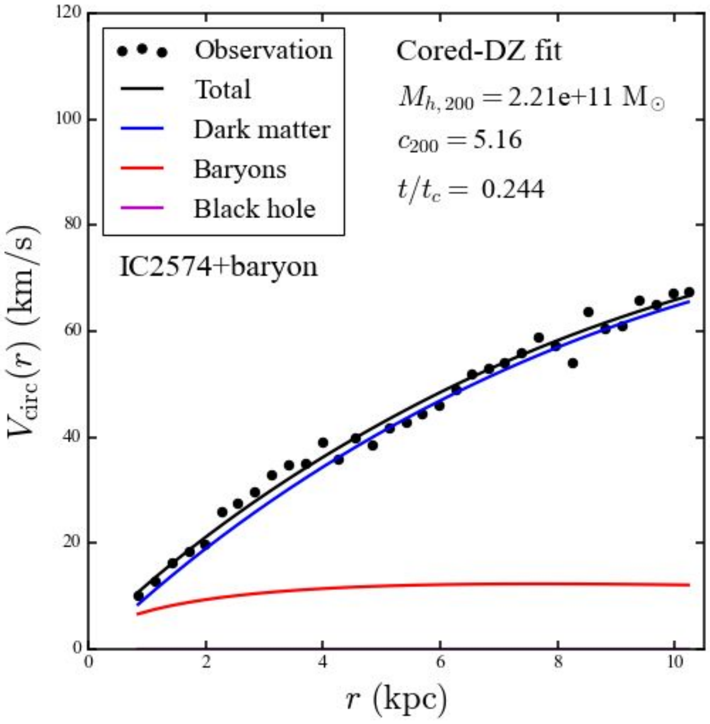
SIDM


Observation
Total
Dark matter
Baryons
Black hole
Cored-DZ fit
$M_{h,200} = 2.21e{+}11\ M_\odot$
$c_{200} = 5.16$
$t/t_c = 0.244$
IC2574+baryon
$V_{circ}(r)$ (km/s)
r (kpc)

DM-only

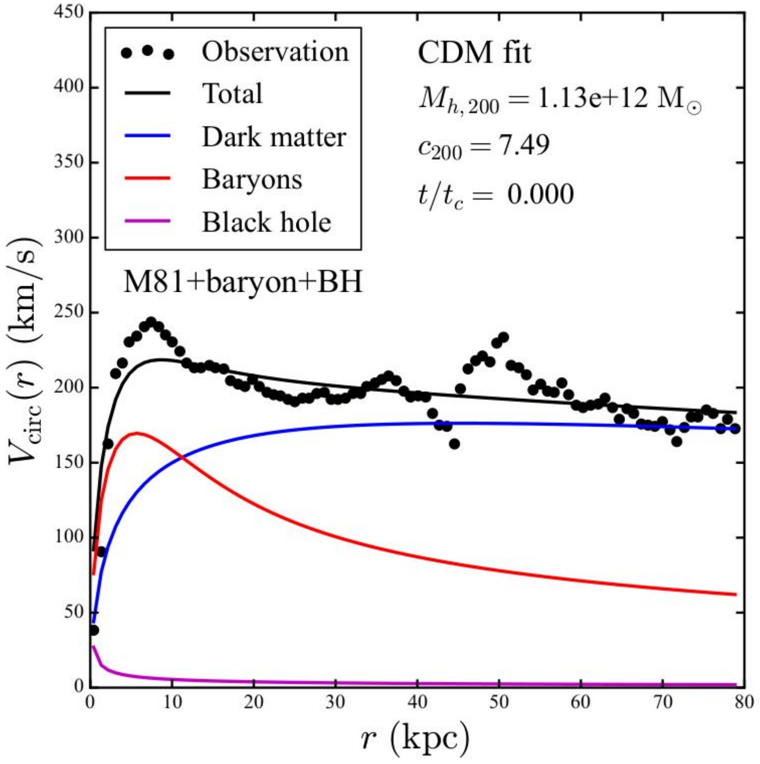


DM+baryons

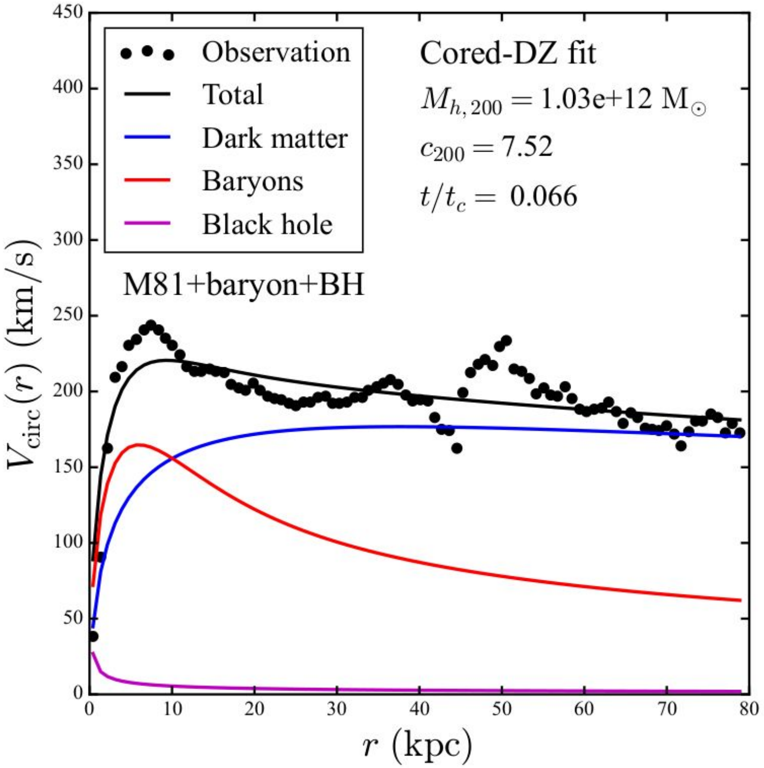


CDM

SIDM



Observation
Total
Dark matter
Baryons
Black hole
CDM fit
$M_{h,200} = 1.13\text{e+}12\ M_\odot$
$c_{200} = 7.49$
$t/t_c = 0.000$
M81+baryon+BH
$V_{circ}(r)$ (km/s)
r (kpc)



Observation
Total
Dark matter
Baryons
Black hole
Cored-DZ fit
$M_{h,200} = 1.03\text{e+}12\ M_\odot$
$c_{200} = 7.52$
$t/t_c = 0.066$
M81+baryon+BH
$V_{circ}(r)$ (km/s)
r (kpc)

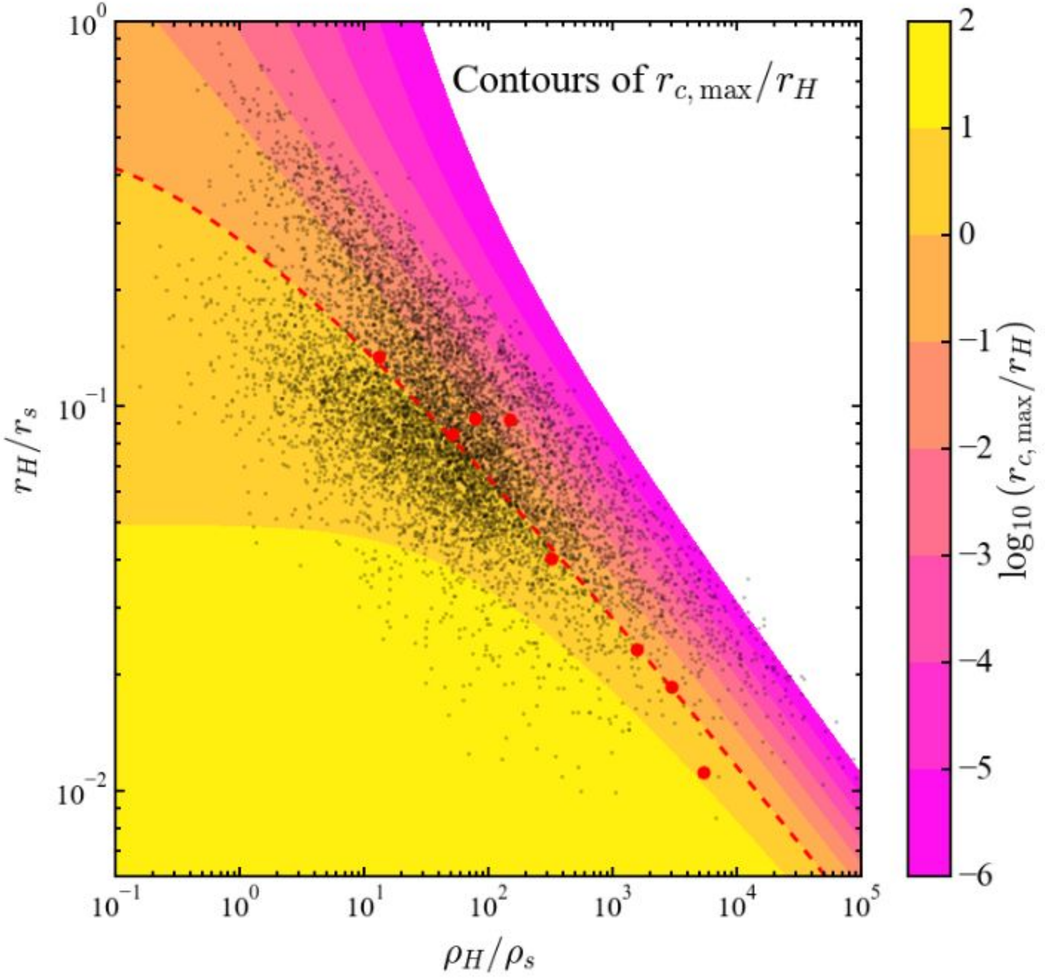
Interplay between the halo and baryon profiles

SIDM core shrinked:

rcmax=0.5 rs (tcb/tc)^2

Lower-left: Baryon may become more diffuse

Upper-right: Small effect during core formation, more compact during core collapse; for both the halo and baryon profiles



Model predicted vs simulated density profiles with baryons

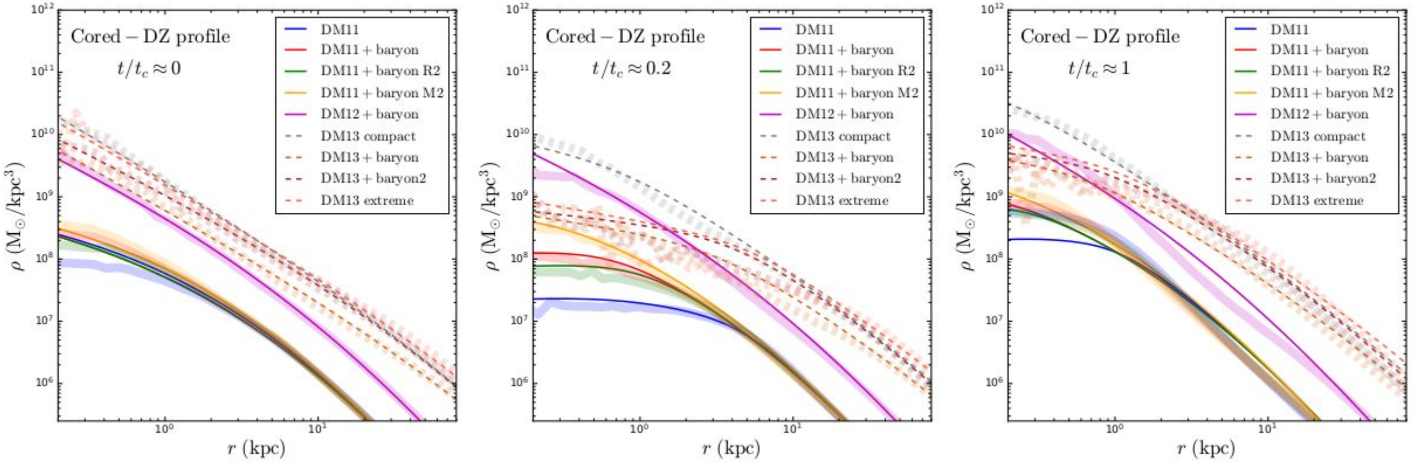


FIG. 5. The simulated (colored bands) and Core-DZ model predicted (colored curves) halo density profiles at three representative gravothermal phases: $t/t_c \approx 0, 0.2$, and 1. The *DM12* and *DM13* scenarios use a contracted CDM profile as the initial condition, whereas the *DM11* scenarios commence with an instant insertion method. In the left panel ($t/t_c \approx 0$), the *DM11* cases are depicted at $t = 0.25$ Gyr to allow some initial evolution away from the original NFW profile. At $t/t_c \approx 1$, the core collapse time, as calculated using Eq. $\boxed{9}$, is found to be 10% (30%) shorter than the simulated *DM13+baryon2* (*DM13 extreme*). To align the profiles for equivalent gravothermal phases, we adjust the timing of the simulated curves accordingly in these specific cases.

Equations

The CoredDZ profile is parameterized as

$$\rho_{\mathrm{CoredDZ}}(r) = \frac{f_{\mathrm{in}}(r)\rho_x f_{\mathrm{out}}(r)}{\frac{(r^k+r_c^k)^{1/k}}{r_x}\left(1+\left(\frac{r}{r_x}\right)^{1/2}\right)^{2(3.5-a)}} \quad (19)$$

where we have introduced two functions to reshape the inner and outer profiles

$$f_{\mathrm{in}}(r) = \left(\frac{r}{r_x} + \frac{r_c}{0.4r_s}\left(\frac{\rho_{x,0}r_{x,0}}{\rho_s r_s + 0.4\rho_H r_H}\right)^{1/(a-1)}\right)^{1-a} \quad (20)$$

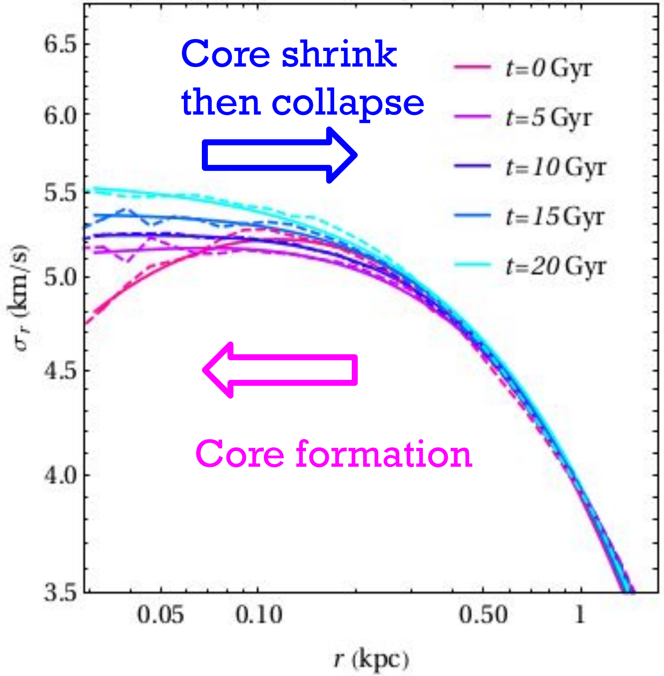
$$f_{\mathrm{out}}(r) = \left(1 + \frac{r}{R_{\mathrm{cut}}}\left(\frac{r_x}{r_{x,0}} - 1\right)\right)^{-1/2}.$$

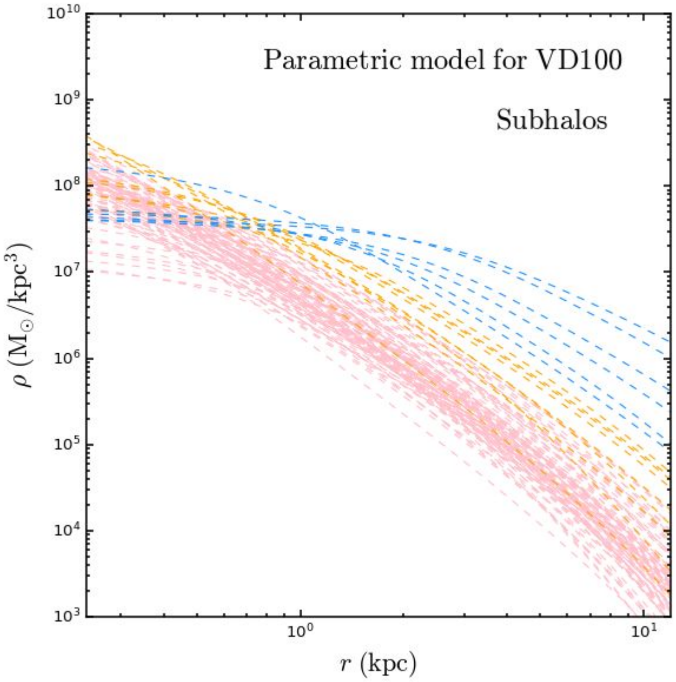
$$\begin{aligned} r_{\mathrm{eff}} &= \frac{r_s\Phi_{0,\mathrm{NFW}} + \alpha r_H \Phi_{\mathrm{Hern}}(0)}{\Phi_{0,\mathrm{NFW}} + \alpha\Phi_{\mathrm{Hern}}(0)} \\ &= \frac{\rho_s r_s^3 + \alpha\rho_H r_H^3/2}{\rho_s r_s^2 + \alpha\rho_H r_H^2/2} \\ &= r_s \frac{1+\alpha\hat{\rho}_H \hat{r}_H^3/2}{1+\alpha\hat{\rho}_H \hat{r}_H^2/2} \equiv r_s \hat{r}_{\mathrm{eff}}, \end{aligned}$$

SIDM enriches inner halo structures

T=m v^2

Velocity dispersion





SIDM leads to a **self-gravitating** & **thermalizing** system

For > 100 Milky Way subhalos
A strong and velocity dependent cross section

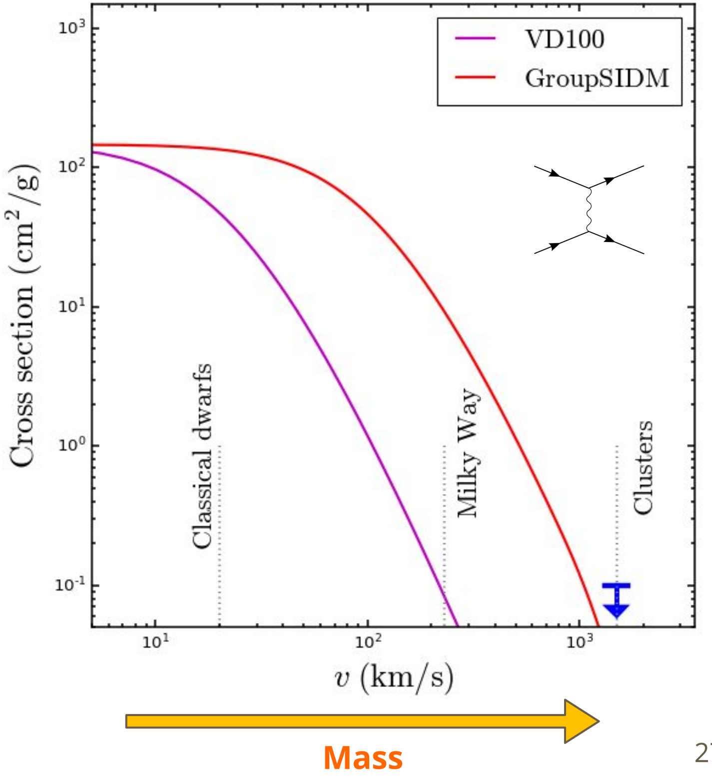
Velocity-dependence accommodate constraints and explain anomalies

$$\frac{d\sigma}{d\cos\theta} = \frac{\sigma_0 w^4}{2\left[w^2 + v^2 \sin^2(\theta/2)\right]^2}$$

Rutherford

For identical particles, consider Moller scatterings;
(JCAP 09 (2022) 077)

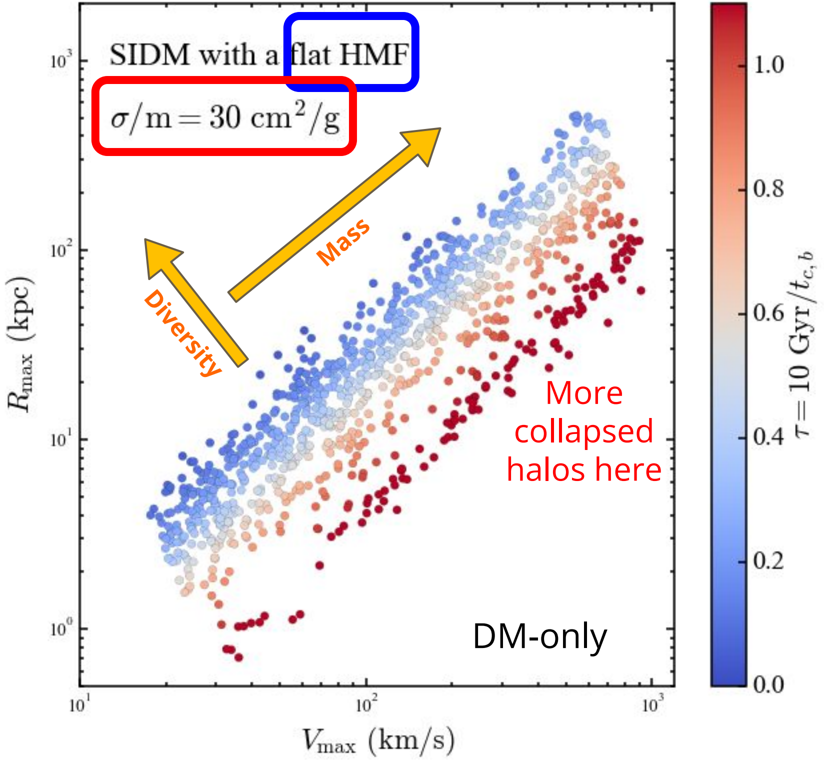
Velocity and angular dependence determined by particle physics models



A constant SIDM cross section does not affect halos in the same way

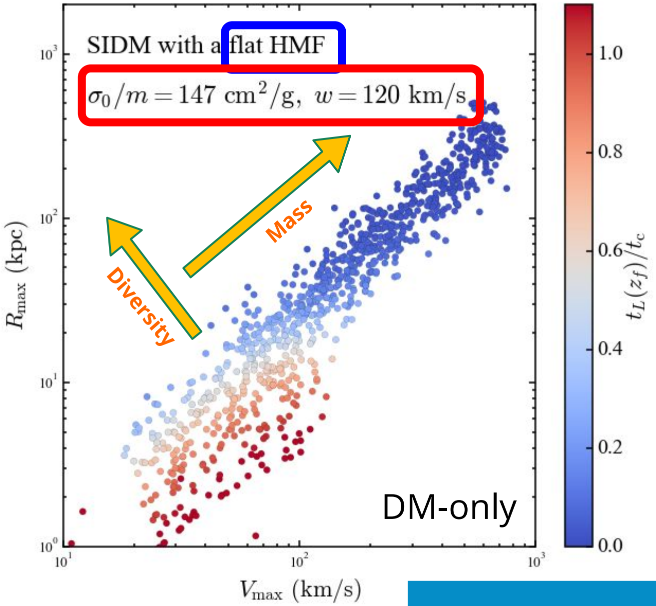
Collisional relaxation

$$t_{c,0} = \frac{150}{C} \frac{1}{\frac{\sigma}{m}\rho_s} \left(\frac{1}{4\pi G \rho_s r_s^2} \right)^{\frac{1}{2}}$$

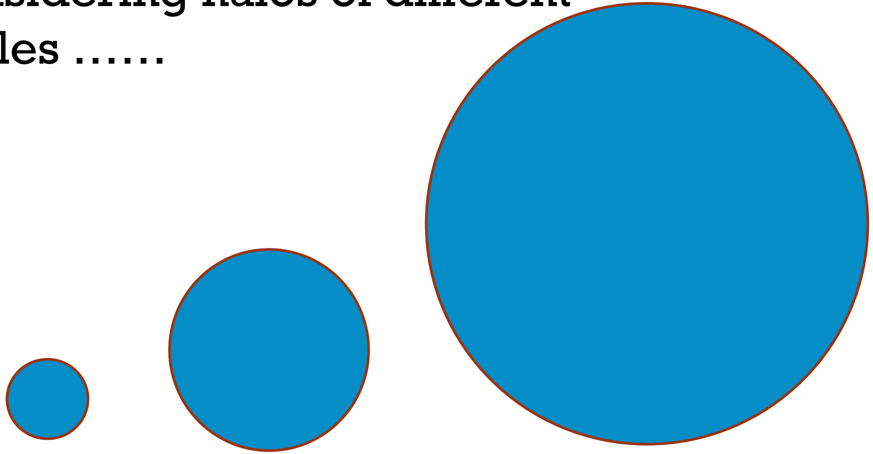


Phys. Rev. Lett. 123, 121102 (2019)
Astrophys. J. 568, 475–487 (2002)

Opportunities



- Rich existing & upcoming observations
- Particle physics scattering information can be recovered by considering halos of different scales ......

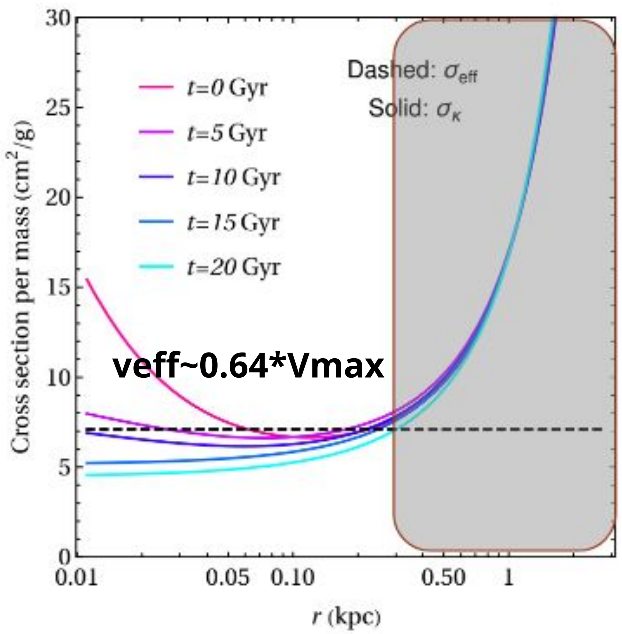


	Halo 1	Halo 2	Halo 3
Model 0	SIDM 1	SIDM 1	SIDM 1
Model 1	SIDM 10	SIDM 1	SIDM 0.1
Model 2	SIDM 100	SIDM 10	SIDM 0.01

One halo probes
One effective constant cross section

Effective constant cross section
(motivated by heat conduction)

$$\sigma_{\text{eff}} = \frac{2\int v^2 dv\, d\cos\theta \boxed{\frac{d\sigma}{d\cos\theta}} \boxed{\sin^2\theta\, v^5} \boxed{\exp\left[-\frac{v^2}{4\nu_{\text{eff}}^2}\right]}}{\int v^2 dv\, d\cos\theta \boxed{\sin^2\theta\, v^5} \boxed{\exp\left[-\frac{v^2}{4\nu_{\text{eff}}^2}\right]}}$$



- Angular dependence is completely integrated out
- Only the ***velocity dependence of SIDM*** couples to the ***halo velocity dispersion***
- *Details of an SIDM model* ***hidden*** *in a single halo*

Thermodynamic equations

$$\text{Continuity equation: } \frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{u}) = 0,$$

$$\text{Jeans equation: } \rho\left(\frac{\partial u_i}{\partial t} + u_j \nabla_j u_i\right) = -\nabla_i P - \nabla_j \Pi_{ij}^{\mathrm{vis}} - \rho \nabla_i \Phi,$$

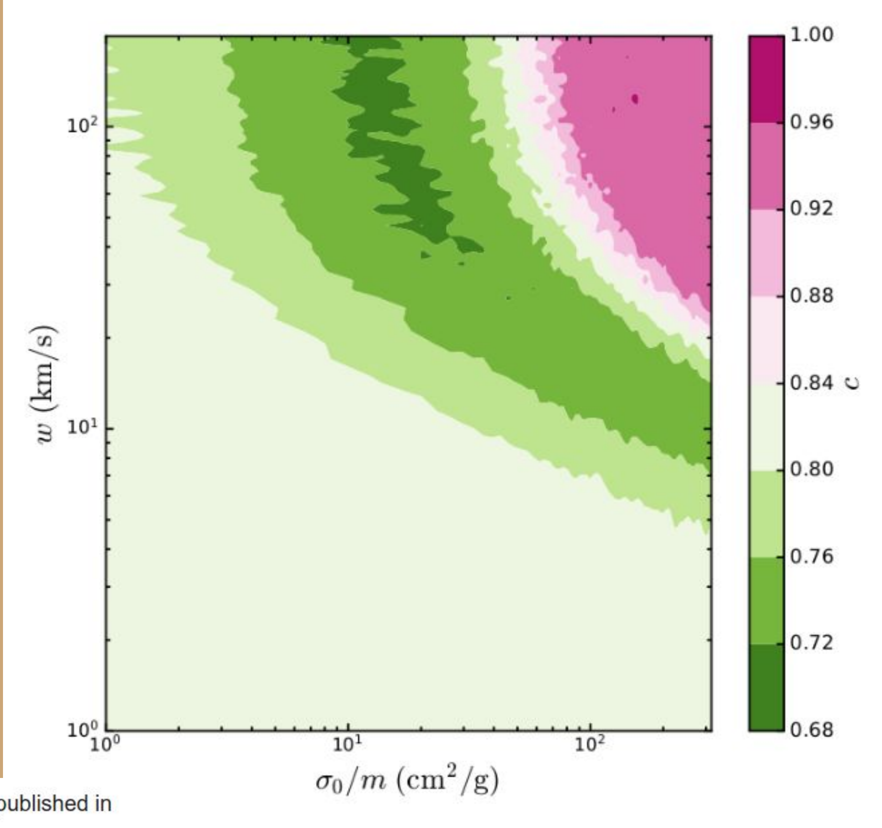
$$\text{Transport equation: } \frac{3\rho}{2m}\left(\frac{\partial T}{\partial t} + \langle v_i \rangle \nabla_i T\right) = -\nabla_i J_i - P \nabla_i \langle v_i \rangle - \Pi_{ij}^{\mathrm{vis}} \partial_i \langle v_j \rangle - \rho \nabla_i \Phi \cdot \langle v_i \rangle\,,$$

where $\mathbf{v} = \mathbf{p}/m = \mathbf{u} + \mathbf{w}$, $\mathbf{u} = \mathbf{p}_c/m = \langle v \rangle$, and $\langle \mathbf{v} \rangle = \frac{1}{\rho} \int d^3 v f \mathbf{v}$.

Applications

- SIDM parameter scan
- Translate CDM simulations into SIDM
- Semi-analytic model/MC program
- Fitting rotation curves
- ……

The efficiency enables exploring parameter space on a laptop



- Dark matter-only version: D. Yang, E. O. Nadler, H.-B. Yu, and Y.-M. Zhong, arXiv:2305.16176, published in JCAP 02, 032 (2024)
- Dark matter plus baryon version: D. Yang, arXiv:2405.03787
- Our method has been implemented in the SASHIMI program for SIDM subhalos: S. Ando, S. Horigome, E. O. Nadler, D. Yang, and H.-B. Yu, arXiv:2403.16633